

# Gravity and the Orbits of Planets

## 1. Gravity

- Galileo
- Newton
- Earth's Gravity
- Mass v. Weight
- Einstein and General Relativity
- Round and irregular shaped objects

## 2. Orbits and Kepler's Laws

ESO

Hubble



CALIFORNIA STATE  
UNIVERSITY  
EAST BAY



## Galileo, Gravity, and a Scientific Revolution

Aristotle believed that objects moved to the center of the geocentric universe because it was their nature. One common understanding regarding his ideas about gravity was that heavier objects would accelerate faster than light objects. Thus, the weight of an object determined how fast it would fall.

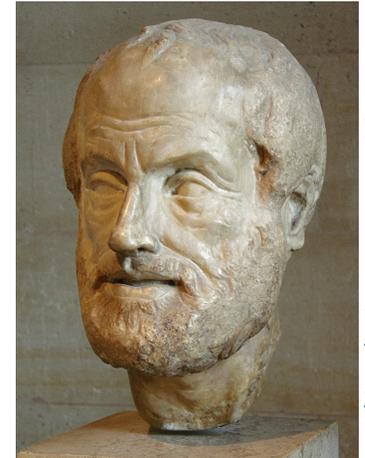


Galileo Galilei began research into our modern understanding about gravity in the late 16<sup>th</sup> century.

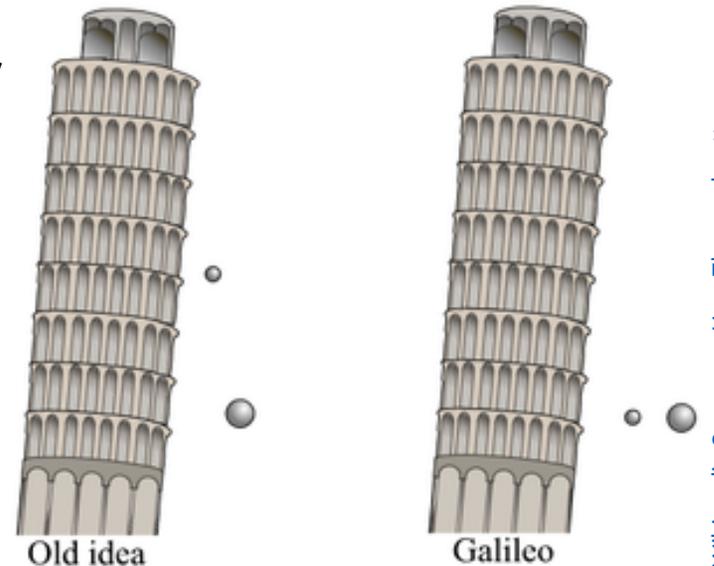
In an experiment commonly attributed to Galileo (but may be apocryphal), he demonstrated that the acceleration of objects was not dependent on the mass of the objects.

Galileo made careful measurements of balls rolling down inclines to prove that the acceleration of an object was independent of its mass.

Galileo observed that lighter objects may fall more slowly but postulated that was due to air resistance (and not to the nature of gravity).



[Wikimedia Commons user: Sting](#)



[Wikimedia Commons User:Theresa knott](#)

## Newton and Universal Gravitation

In 1687, Sir Isaac Newton published his work *Principia* about the laws of motion and gravitation. In it, he hypothesized the inverse-square law of universal gravitation

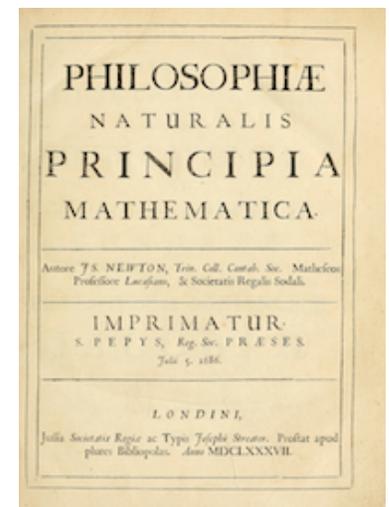
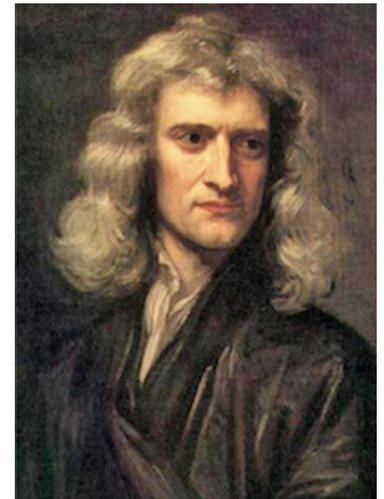
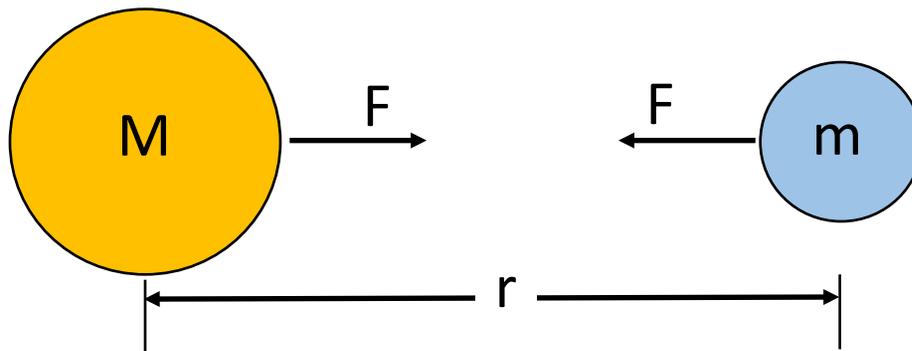
$$F = G \left( \frac{Mm}{r^2} \right)$$

Where force,  $F$ , is proportional to the masses of two objects ( $M$ ,  $m$ ) and inversely proportional to the square of the distance between them ( $r$ ). The universal gravitational constant,  $G$ , is an empirical physical constant

$$G = 6.674 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2} = 6.674 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$$

Every object attracts every other object

- the closer the objects are, the greater the force
- the more massive the objects, the greater the force



## The Earth's Gravity: $G$ versus $g$

Newton's second law of gravity describes the relationship between mass and the amount of force ( $F$ ) that is required to accelerate it.

$$F = ma$$

This means that the more mass that an object has, the more force is required to accelerate it.

Combining Newton's laws

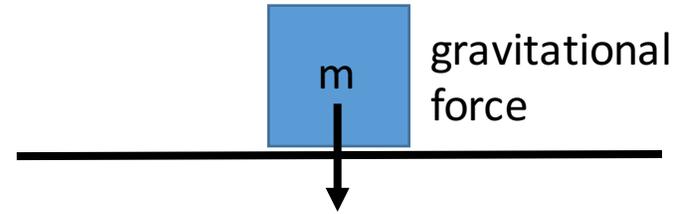
$$F = G \left( \frac{Mm}{r^2} \right) = ma$$

Note that  $m$  occurs on both sides of the equation and can be cancelled. Rearranging

$$a = \frac{GM}{r^2}$$

where  $M$  is the mass of the Earth. At the Earth's surface,  $r$  is the Earth's radius.

$$\begin{aligned} a &= \frac{(6.674 \cdot 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2)(5.98 \cdot 10^{24} \text{ kg})}{(6.38 \cdot 10^6 \text{ m})^2} \\ &= 9.8 \text{ m/sec}^2 \end{aligned}$$



The gravity on Earth is commonly noted as  $g$  and refers to the acceleration that the Earth imparts on objects on or near its surface due to gravity. This quantity is informally called “little  $g$ ” the gravitational constant,  $G$ , is informally call “big  $G$ .” On Earth, Newton’s second law can be rewritten

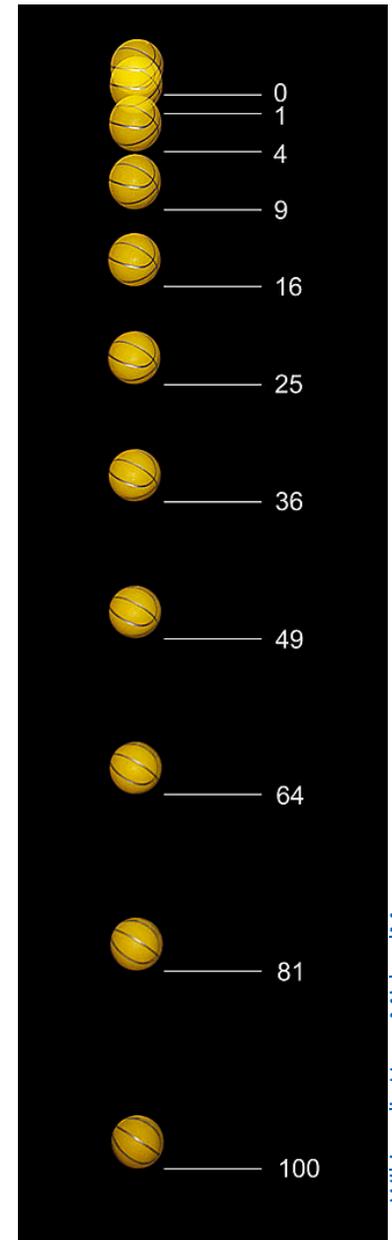
$$F = ma = mg$$

The acceleration due to the Earth’s gravity is  $\sim 9.8 \text{ m/s}^2$ . That means that the *speed* of an object falling near the Earth's surface will increase by about 9.8 meters ( $\sim 32 \text{ ft}$ ) per second every second.

For a satellite in orbit around the Earth, the value of  $g$  is less since it is farther from the Earth’s center.

Location	Value of $g$ ( $\text{m/s}^2$ )
Earth’s surface	9.8
1000 km above surface	7.33
10,000 km above surface	1.49
50,000 km above surface	0.13

The value of  $g$  decreases rapidly with distance from the Earth due to the inverse square relationship with distance.



The same equation can be used to determine the gravitational acceleration on other solar system bodies.

$$g_{\text{planet}} = \frac{GM}{r^2}$$

where  $M$  is the mass of a planet and  $r$  is the radius of the planet.

Note that the value of  $g$  is independent of the mass of an object but is only dependent on the mass of the planet and the location of the object (distance from the center of the planet).

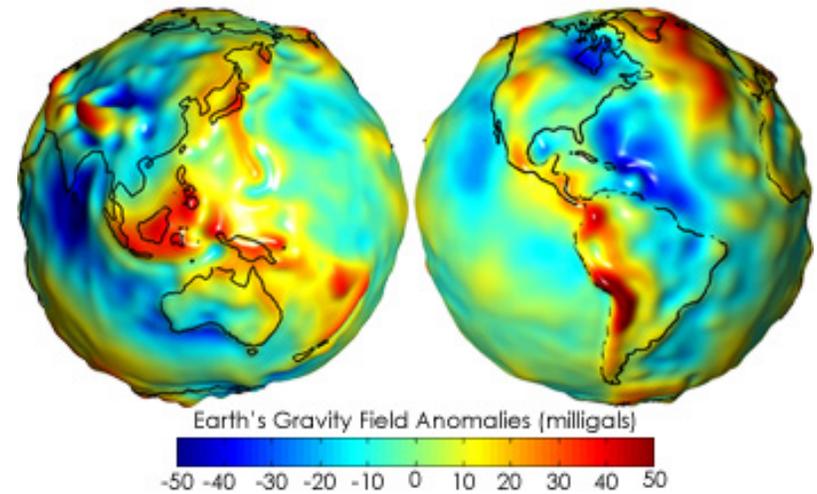
Planet	Radius (m)	Mass (kg)	$g$ (m/s <sup>2</sup> )	$g_{\text{Earth}}/g_{\text{Planet}}$
Mercury	$2.43 \times 10^6$	$3.2 \times 10^{23}$	3.61	0.37
Venus	$6.07 \times 10^6$	$4.88 \times 10^{24}$	8.83	0.90
Earth	$6.37 \times 10^6$	$5.97 \times 10^{24}$	9.81	1
Mars	$3.38 \times 10^6$	$6.42 \times 10^{23}$	3.75	0.38
Jupiter	$6.98 \times 10^7$	$1.90 \times 10^{27}$	26.0	2.65
Saturn	$5.82 \times 10^7$	$5.68 \times 10^{26}$	11.2	1.14
Uranus	$2.35 \times 10^7$	$8.68 \times 10^{25}$	10.5	1.07
Neptune	$2.27 \times 10^7$	$1.03 \times 10^{26}$	13.3	1.35
Pluto	$1.15 \times 10^6$	$1.2 \times 10^{22}$	0.61	0.06

## Variations in the Earth's Gravitation Acceleration

The average value of  $g$  at the Earth's surface is  $9.8 \text{ m/s}^2$ .

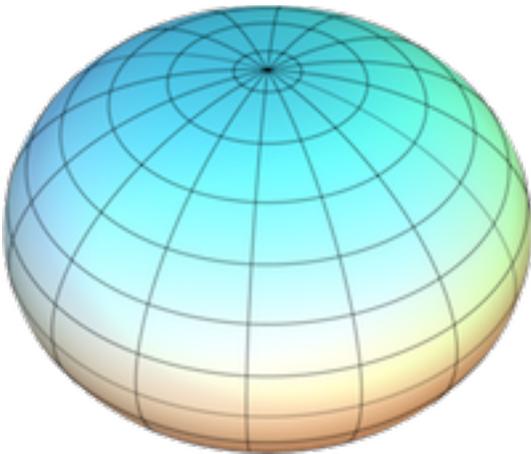
There are some local variations that result from the density of the geologic structures and rocks below the surface.

The animation shows gravity variations where red indicates where the Earth's gravity is stronger and blue where Earth's gravity is weaker



GRACE/NASA

[Wikimedia User: AugPi](#)



Variations in the Earth's gravity also result from the fact that the Earth is not perfectly spherical (oblate spheroid) but is slightly flattened at the poles. Since the earth's surface at the poles is slightly closer to the center of the Earth, the value of  $g$  is greater at the poles.

These variations in the value of  $g$  are less than 0.7% and range from  $9.7639\text{-}9.8337 \text{ m/s}^2$ .

Small variations in the Earth's gravity has become a valuable geophysical technique for inferring structures in the interior and for prospecting for mineral resources.

## Mass v. Weight

Another implication of Newton's second law is that the force,  $F$ , is an object's weight.

$$F = ma = mg$$

- *Mass* is a measure of the amount of matter in an object.
- *Weight* is the force experienced by an object due to gravity.

On Earth, a 1 kg object weighs

$$F = \text{weight} = (1 \text{ kg})(9.81 \text{ m/s}^2) = 9.81 \text{ Newtons} = 2.2 \text{ pounds}$$

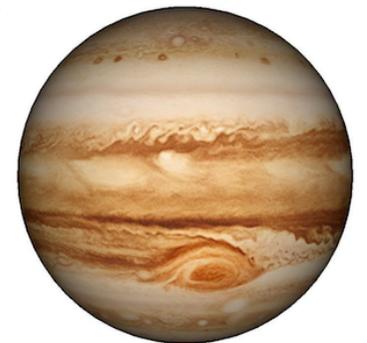
However, a 1 kg object would have a different weight on a different planet.

On Pluto, a 1 kg object would weigh

$$F = \text{weight} = mg_{\text{pluto}} = (1 \text{ kg})(0.61 \text{ m/s}^2) = 0.61 \text{ Newtons} = 0.14 \text{ pounds}$$

On Jupiter, a 1 kg object would weigh

$$F = \text{weight} = mg_{\text{Jupiter}} = (1 \text{ kg})(26.0 \text{ m/s}^2) = 26 \text{ Newtons} = 5.84 \text{ pounds}$$

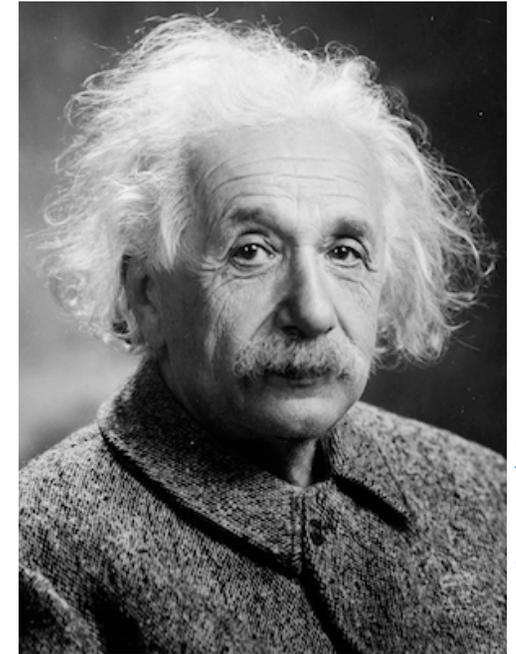


## General relativity

In Einstein's theory of general relativity, he proposed that *spacetime* is curved by matter – massive objects such as stars and planets curve space. Thus objects accelerated by gravity move along paths in curved *spacetime*.

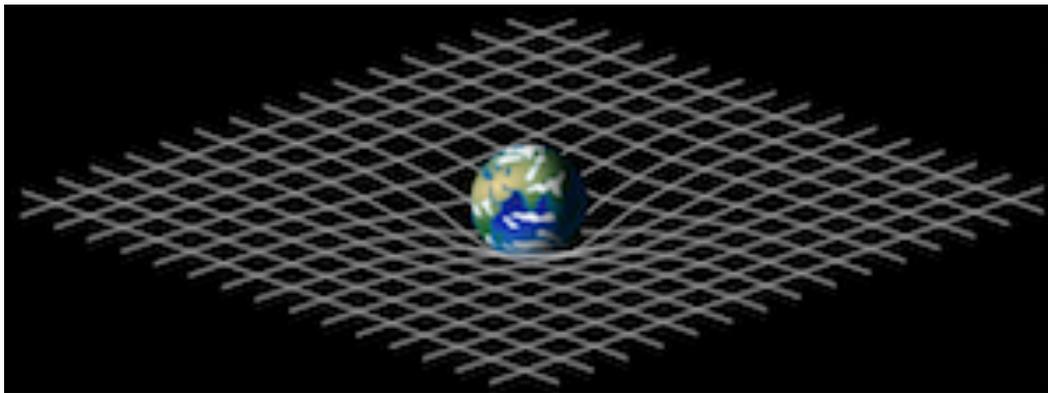
Although more precise, solutions to Einstein's equations are more difficult than classical Newtonian gravitation.

Newtonian mechanics is sufficiently accurate and is used for launching rockets and satellites.

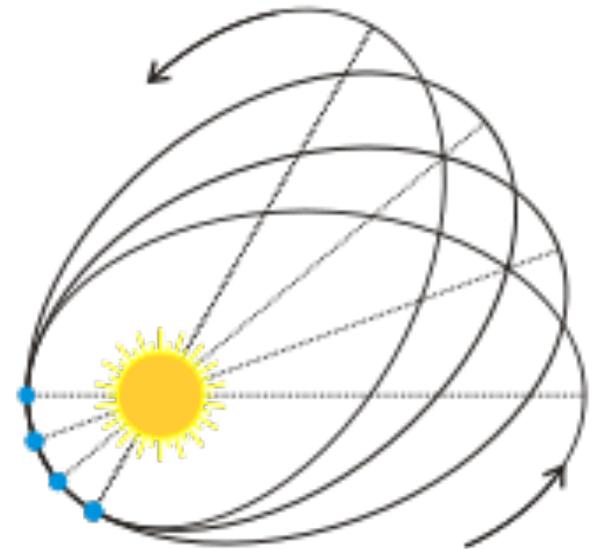


[Orren Jack Turner](#)

[Wikimedia User: Mysid](#)



Proof of Relativity: The orbit of Mercury has been observed to change (precession) slightly with time and cannot be described by Newtonian mechanics. However, relativity accurately predicts the orbit of Mercury.



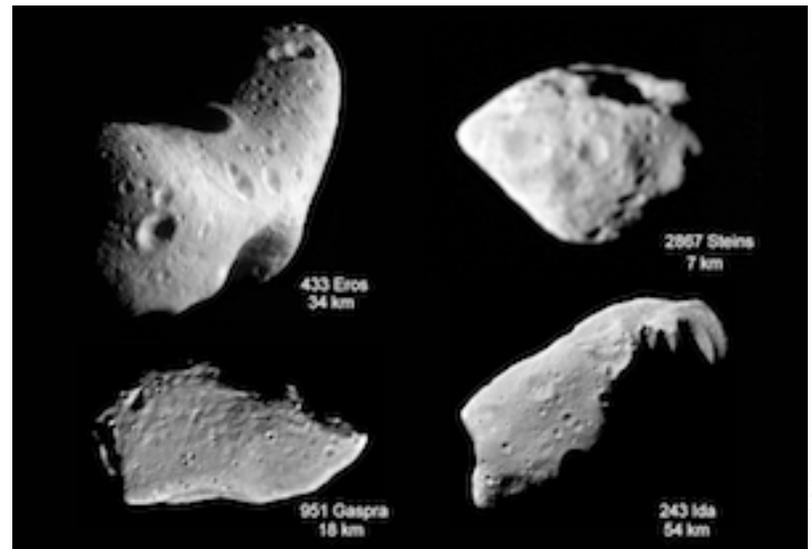
[Wikimedia User: Mpfiz](#)

## Rounding of Planets and Moons

Planets and large moons are round because their gravitational field is directed to the center of the body and pulls everything toward it. With large bodies, internal heating causes it to behave somewhat like a fluid and form a sphere. A sphere is a shape with the least amount of surface area.

In small bodies such as asteroids, the gravitational pull is too weak to overcome the mechanical strength of the asteroid. As a result, these bodies do not form spheres and maintain irregular shapes.

Objects with a diameter  $>1000$  km tend to form spherical shapes whereas small objects tend to form irregular shapes.

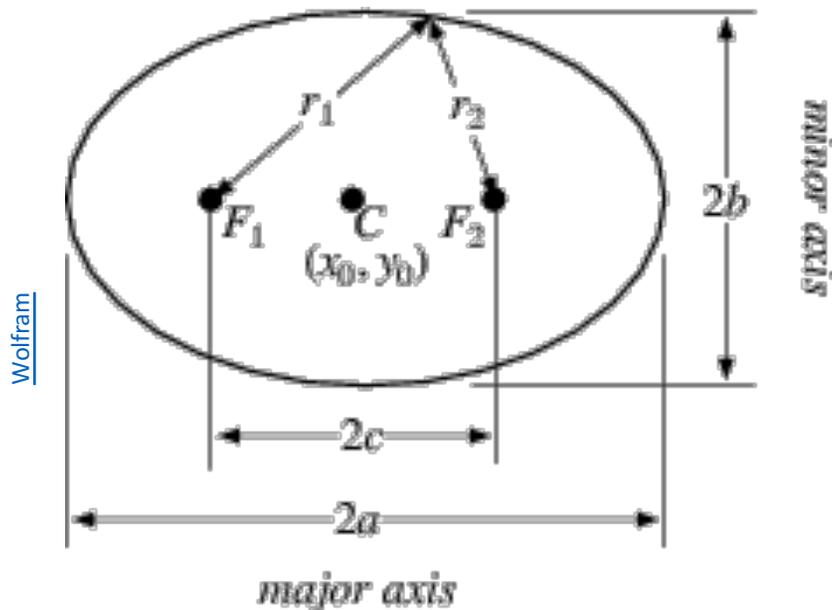


## 2. Kepler and the Laws of Planetary Motion

Johannes Kepler was a German mathematician and astronomer that studied planetary motion and proposed three laws that govern the orbital relationships of planets in the 17<sup>th</sup> century.

**Kepler's First Law:** The orbit of a planet is an ellipse with the Sun at one of the two foci (Law of Ellipses).

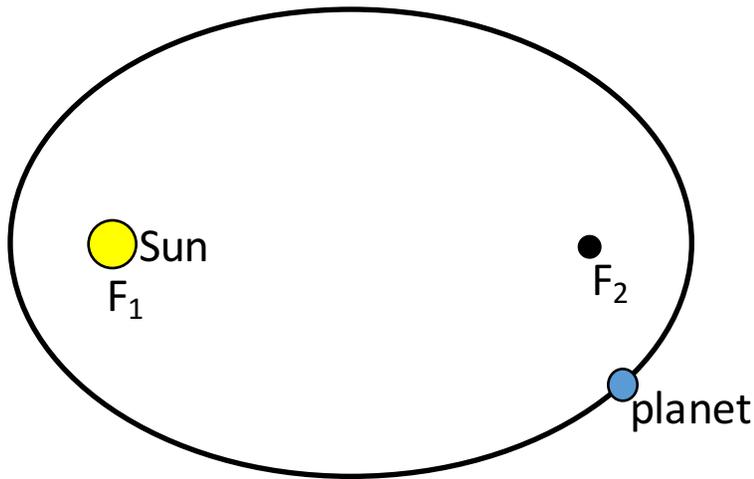
This relationship also governs the orbits of moons around planets.



An *ellipse* is a closed curve that surrounds two focal points such that the sum of the distances to the two focal points is constant for every point on the curve.

The closer the foci are to one another, the more it resembles a circle. A circle is a special case of an ellipse where the two foci are at the same location.

The distance between the two foci affect the shape of the ellipse. The greater the distance between the foci, the more elongate the ellipse. This is known as *eccentricity*.



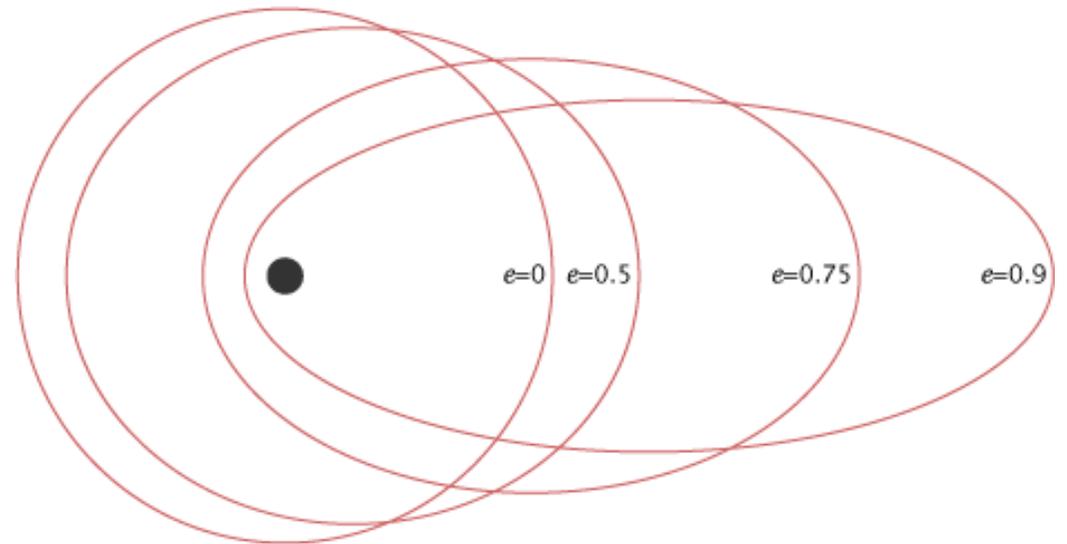
Although planetary orbits are nearly circular, they are more precisely described as ellipses.

The Sun is located at one focus.

The second focus is in empty space and has no significance.

*Eccentricity exaggerated*

Solar System Objects	Orbital Eccentricity
Mercury	0.206
Venus	0.007
Earth	0.017
Mars	0.093
Jupiter	0.048
Saturn	0.056
Uranus	0.046
Neptune	0.010
Pluto	0.248
Halley's Comet	0.967



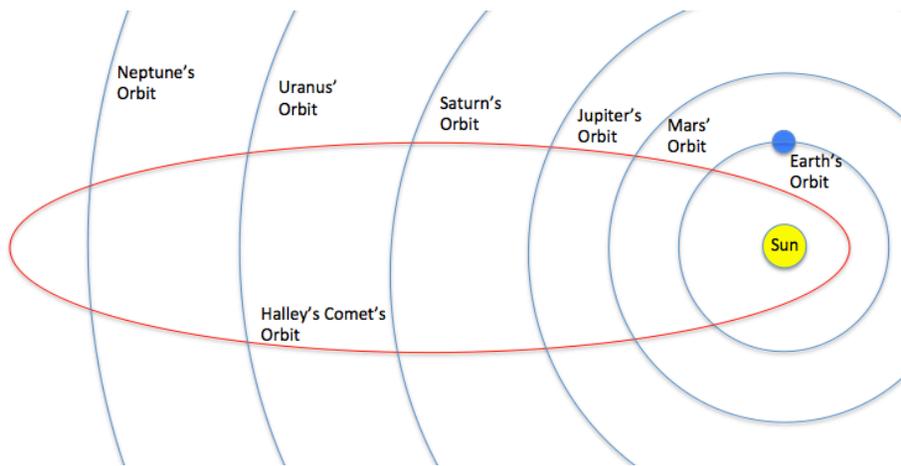
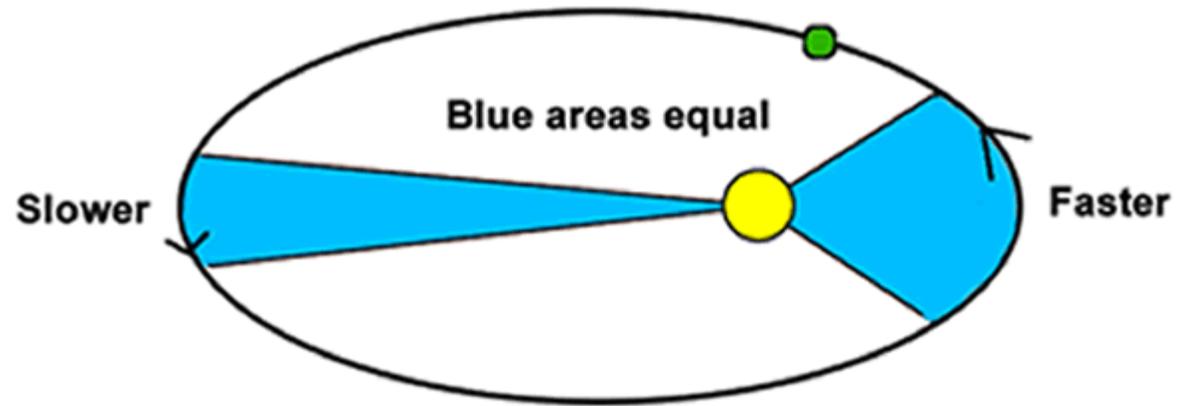
NASA

The eccentricity factor for an ellipse ranges from zero to less than one.

Eccentricity factors greater than or equal to one describe parabolas and hyperbolas.

**Kepler's Second Law:** A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time (Law of Equal Areas).

This law describes the speed of an object during its orbit around the Sun; the object has a greater orbital velocity when it is closer to the Sun and slows as it moves away from the Sun.



Halley's comet has a highly eccentric orbit with an eccentricity factor of 0.967.

The table shows that the velocity of Halley's comet increases significantly during its closest approach to the Sun (perihelion).

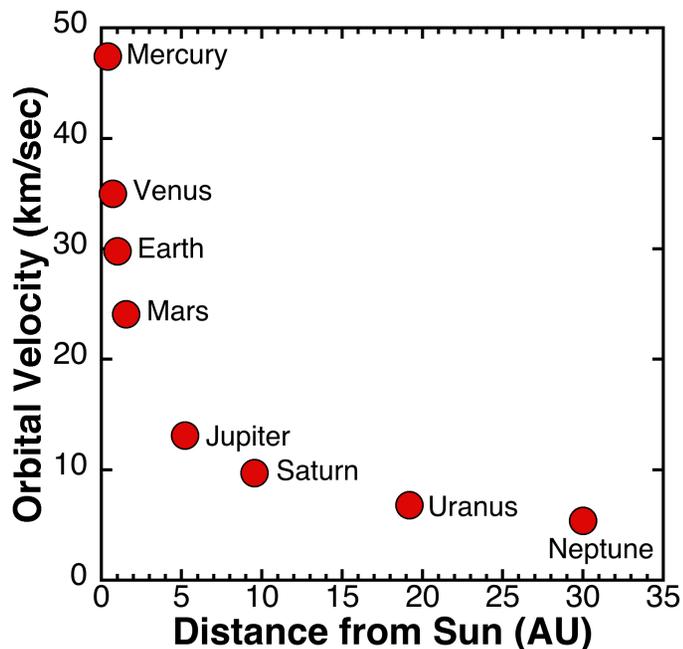
	Perihelion	Aphelion
Distance from Sun	0.587 AU	35.11 AU
Orbital velocity	54,600 m/s	900 m/s

**Kepler's Third Law:** The ratio of the squares of the periods ( $P$ ) of any two planets is proportional to the ratio of the cubes of their average distances ( $d$ ) from the Sun. (Law of Harmonies).

$$\frac{P_1^2}{P_2^2} = \frac{d_1^3}{d_2^3}$$

where  $d$  is the distance from the Sun and  $p$  is orbital period

This law describes the relationship between the distance of a planet from the Sun ( $d$ ) and its orbital period ( $P$ ) and implies that the closer a planet is to the Sun, the greater its speed in its orbit around the Sun.

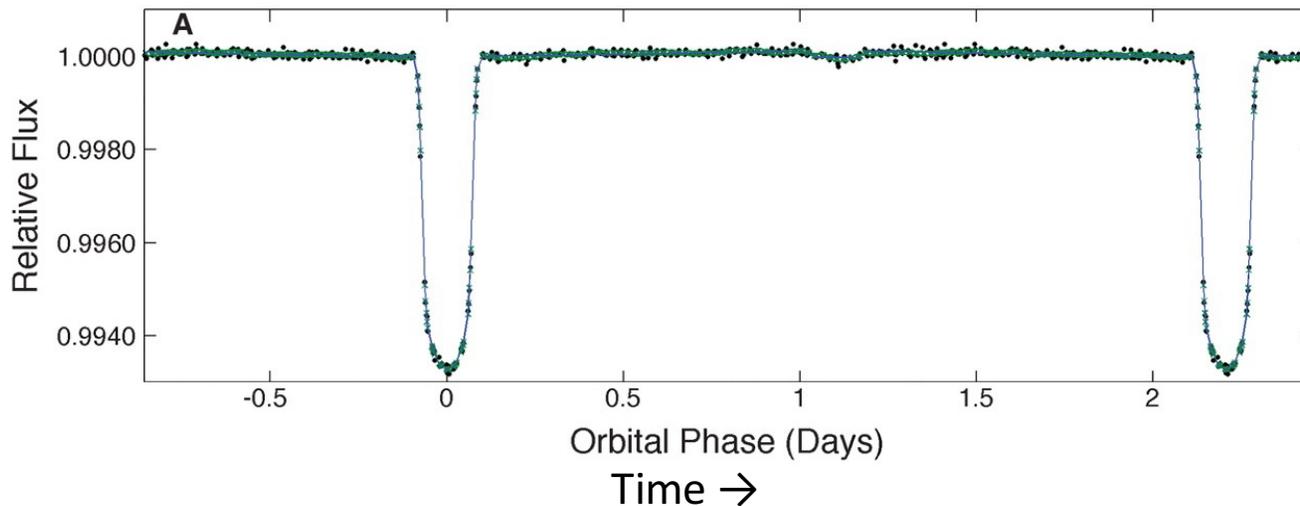


Planet	Period (p) Year	Distance (d) AU	$d^3/P^2$	Orbital Velocity (km/s)
Mercury	0.241	0.387	0.98	47.4
Venus	0.616	0.723	1.01	35.0
Earth	1	1	1	29.8
Mars	1.88	1.524	1.01	24.1
Jupiter	11.9	5.203	0.99	13.1
Saturn	29.5	9.539	1.00	9.7
Uranus	84.0	19.191	1.00	6.8
Neptune	165.0	30.071	1.00	5.4

## Significance of Kepler's Laws: Detection of Exoplanets

Kepler is a space observatory launched in 2009 by NASA to discover Earth-like planets orbiting other stars.

Kepler was designed to observe a portion of our region of the Milky Way. Kepler's primary instrument was a photometer that continually monitored the brightness of over 145,000 main sequence stars as exoplanets pass in front of them.



If we observe an exoplanet passing in front of a star more than once, we can determine its orbital period.

Then we can use Kepler's third law to estimate the distance of the exoplanet from the star.